

Massless DKP fields in Riemann-Cartan space-times

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Abstract

We study massless Duffin-Kemmer-Petiau (DKP) fields in the context of Einstein-Cartan gravitation theory, interacting via minimal coupling procedure. In the case of an identically vanishing torsion (Riemannian space-times) we show that there exists local gauge symmetries which reproduce the usual gauge symmetries for the massless scalar and electromagnetic fields. On the other hand, similarly to what happens with the Maxwell theory, a non-vanishing torsion, in general, breaks the usual $U(1)$ local gauge symmetry of the electromagnetic field or, in a different point of view, impose conditions on the torsion.

1 Introduction

The *Duffin-Kemmer-Petiau equation* (DKP) is a first order relativistic wave equation that describes spin 0 and 1 fields [1, 2, 3]. It is analogous to the Dirac equation, but differs by the algebra satisfied by the β^μ matrices, correspondent to the Dirac γ^μ ones, which have only three irreducible representations of dimensions 1 (trivial), 5 (spin 0) and 10 (spin 1).

In the last years there have been a renewed interest in DKP theory as it was realized that DKP theory is richer than the KG one with respect to the introduction of interactions [5, 6]. For instance, it has been studied in the context of QCD [7], covariant hamiltonian dynamics [8], in the causal approach [9], in the context of five-dimensional galilean covariance [10], in the scattering K^+ -nucleus [11], etc. On the other hand, there have been some efforts to give strict proofs of equivalence between the KG equation and the spin 0 sector of DKP equation in various situations [12, 13, 14]. In the same context, some aspects regarding the minimal interaction with the electromagnetic field have been clarified [15, 16].

In the context of curved space-times, it was proved the complete equivalence between DKP and KG and Proca fields in a riemannian space-time [17, 18]. Moreover, it was shown that in the context of Einstein-Cartan gravity DKP theory naturally induces an interaction between the spin 0 field and the space-time torsion, which breaks the equivalence with the KG theory [19]. Besides that, DKP theory induces

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additional couplings between spin 1 fields and the space-time torsion, what also breaks the equivalence with the Proca theory [20].

In the present work we develop further the above analysis by studying the *massless* DKP field minimally coupled to Riemann-Cartan space-times, as well the gauge invariance properties of the theory. It is important to notice that the massless case can not be obtained through the limit $m \rightarrow 0$ of the massive case. This is due to the fact that the projections of DKP field into spin 0 and 1 sectors involve the mass as a multiplicative factor [19, 20] so that taking the limit $m \rightarrow 0$ makes the results previously obtained useless. Moreover, if we simply make mass equal to zero in the usual massive DKP Lagrangian we obtain a Lagrangian with no local gauge symmetry. As will be seen, the solution is to change the usual mass term in the DKP Lagrangian to a term containing a singular matrix, what will change the way we manipulate the equation of motion.

So, this paper is organized as follows: in section 2 we present the formalism introduced by Harish-Chandra for the massless DKP theory in the Minkowski space-time [21] and select its spin 0 and 1 sectors through the use of the Umezawa's projectors [22, 15, 17]. In section 3 we study massless DKP fields minimally coupled to Riemann-Cartan space-times and compare the results with those obtained in the context of KG and Maxwell theories in the presence or not of torsion. In section 4 we present our remarks and conclusions. The basic aspects and properties of DKP equation which are necessary to read this work can be found in the references [17, 15], where it was used the same metric signature ($\eta = \text{diag}(+, -, -, -)$) used here.

2 The massless DKP theory

As mentioned above, massless DKP theory can not be obtained as a zero mass limit of the massive DKP case, so we consider the Harish-Chandra Lagrangian density for the massless DKP theory in the Minkowski space-time \mathcal{M}^4 , given by [21]

$$\mathcal{L}_{\mathcal{M}} = i\bar{\psi}\gamma\beta^a\partial_a\psi - i\partial_a\bar{\psi}\beta^a\gamma\psi - \bar{\psi}\gamma\psi, \quad (1)$$

where the β^a matrices satisfy the usual DKP algebra

$$\beta^a\beta^b\beta^c + \beta^c\beta^b\beta^a = \beta^a\eta^{bc} + \beta^c\eta^{ba} \quad (2)$$

and γ is a *singular* matrix satisfying¹

$$\beta^a\gamma + \gamma\beta^a = \beta^a \quad \text{and} \quad \gamma^2 = \gamma. \quad (3)$$

From the above lagrangian follows the massless DKP wave equation

$$i\beta^a\partial_a\psi - \gamma\psi = 0. \quad (4)$$

The equations (1)-(4), as it was shown by Harish-Chandra, describe in fact four massless gauge theories, which correspond to a massless scalar field, a spin 1 (i.e. electromagnetic) field, a second-rank antisymmetric potential and a third rank linear potential. The last two ones propagate no degree of freedom and are topological field theories [21, 23, 24].

The Lagrangian density (1) and equation (4) are both invariant under the following local gauge transformation

$$\psi \rightarrow \psi' = \psi + (1 - \gamma)\Phi, \quad (5)$$

¹We choose a representation in which $\beta^{0\dagger} = \beta^0$, $\beta^{i\dagger} = -\beta^i$ and $\gamma^\dagger = \gamma$.

where the field Φ is assumed to satisfy the condition

$$i\beta^a\partial_a(1-\gamma)\Phi=0. \quad (6)$$

This condition, when projected to the spin 0 and 1 sectors reproduce, respectively, the global and local $U(1)$ gauge symmetry of the massless Klein–Gordon and electromagnetic fields, as we will show.

2.1 Spin 0 sector

To select the spin 0 sector from equation (4) we use the Umezawa’s projectors P and P^a [22, 15], remembering that under proper Lorentz transformations $P\psi$ transforms as a scalar field while $P^a\psi$ transforms as a vector field. Applying these projectors on equation (4), and taking into account that relations (3) imply $\gamma P = P\gamma$ and $P^a\gamma + \gamma P^a = P^a$, we obtain the equation of motion for the massless scalar field $P\psi$

$$\partial_a\partial^a(P\psi)=0. \quad (7)$$

In terms of the scalar field $P\psi$ the gauge transformation (5) reads

$$P\psi' = P\psi + (1-\gamma)P\Phi \quad , \quad P^a\psi' = P^a\psi + P^a(1-\gamma)\Phi, \quad (8)$$

and condition (6) becomes

$$\partial_aP^a(1-\gamma)\Phi=0 \quad , \quad \partial^aP(1-\gamma)\Phi=0. \quad (9)$$

Therefore, it can be easily verified that equation (7) is invariant under transformation (8).

The results above are independent of the representation for the algebra (2)–(3). Nevertheless, to study the gauge invariance in more details we shall use a specific representation for β^a in which

$$\gamma = \text{diag}(\lambda, 1-\lambda, 1-\lambda, 1-\lambda, 1-\lambda). \quad (10)$$

In this representation the one-column DKP wave function and its projections are given by (with $a = 0, 1, 2, 3$)

$$\psi = \begin{pmatrix} \varphi \\ \psi^a \end{pmatrix}, \quad P\psi = \begin{pmatrix} \varphi \\ [0]_{4 \times 1} \end{pmatrix}, \quad P\gamma\psi = \begin{pmatrix} \lambda\varphi \\ [0]_{4 \times 1} \end{pmatrix}, \quad (11)$$

$$P^a\psi = \begin{pmatrix} \psi^a \\ [0]_{4 \times 1} \end{pmatrix}, \quad P^a\gamma\psi = \begin{pmatrix} (1-\lambda)\psi^a \\ [0]_{4 \times 1} \end{pmatrix} \quad (12)$$

The condition $\gamma^2 - \gamma = 0$ will imply for the λ parameter

$$\lambda^2 - \lambda = 0 \rightarrow \lambda = 0, 1. \quad (13)$$

The value $\lambda = 1$ corresponds to a topological field, while $\lambda = 0$ reproduces the massless Klein–Gordon field [21], as we shall see in the following. In this representation the explicit relations among the components of the massless spin 0 DKP field are

$$(1-\lambda)\psi^a = i\partial^a\varphi, \quad (14)$$

$$\lambda\varphi = i\partial_a\psi^a,$$

where $a = 0, 1, 2, 3$. If the column matrix Φ is given by

$$\Phi = (\varphi_\Phi, \phi^0, \phi^1, \phi^2, \phi^3)^T \quad (15)$$

the gauge transformation (8) for the components of ψ reads

$$\begin{aligned}\varphi' &= \varphi + (1 - \lambda)\varphi_{\Phi}, \\ \psi'^a &= \psi^a + \lambda\phi^a,\end{aligned}\tag{16}$$

while the condition (9) gives

$$\lambda\partial_a\Phi^a = 0 \quad , \quad (1 - \lambda)\partial^a\varphi_{\Phi} = 0.\tag{17}$$

Using the results of the $\lambda = 0$ case it can be seen that the DKP Lagrangian density (1) reduces to the usual one for the massless Klein-Gordon field

$$\mathcal{L}_{\mathcal{M}_{s=0}} = \partial^{\mu}\varphi^*\partial_{\mu}\varphi,\tag{18}$$

which, together with the equation of motion $\partial_a\partial^a\varphi = 0$, is invariant under the gauge transformation $\varphi' = \varphi + \varphi_{\Phi}$, with constant φ_{Φ} .

From equations (14) we see that the $\lambda = 1$ case corresponds to a constant field φ (thus propagating no degree of freedom). This is a topological field which will not be considered here.

2.2 Spin 1 sector

To select the spin 1 sector we use the projectors R^a and R^{ab} , such that under proper Lorentz transformations $R^a\psi$ and $R^{ab}\psi$ transform, respectively, as a vector and a second rank tensor field [22, 15]. Again, applying these projectors on (4) and taking into account that, due to relations (3), we have $\gamma R^a = R^a\gamma$ and $R^{ab}\gamma + \gamma R^{ab} = R^{ab}$ we obtain the equation for the massless vector field $R^a\psi$

$$\partial_b [\partial^a(R^b\psi) - \partial^b(R^a\psi)] = 0.\tag{19}$$

The gauge transformation (5) and the condition (6) yield

$$R^a\psi' = R^a\psi + R^a(1 - \gamma)\Phi \quad , \quad R^{ab}\psi' = R^{ab}\psi + R^{ab}(1 - \gamma)\Phi,\tag{20}$$

$$\partial_b R^{ab}(1 - \gamma)\Phi = 0 \quad , \quad \partial^a R^b(1 - \gamma)\Phi - \partial^b R^a(1 - \gamma)\Phi = 0,\tag{21}$$

and the invariance of (19) under the gauge transformation (20)-(21) can be promptly verified.

Again we shall study the gauge transformation by considering a specific spin 1 representation for β^a such that

$$\gamma = \text{diag}(\lambda, \lambda, \lambda, \lambda, 1 - \lambda),\tag{22}$$

and the condition $\gamma^2 - \gamma = 0$ once more implies that for the λ parameter we have

$$\lambda^2 - \lambda = 0 \rightarrow \lambda = 0, 1.\tag{23}$$

In this representation the wave function ψ and its projections are given by

$$\psi = \begin{pmatrix} [\psi^a]_{4x1} \\ [\psi^{ab}]_{6x1} \end{pmatrix} \quad , \quad R^a\psi = \begin{pmatrix} \psi^a \\ [0]_{9x1} \end{pmatrix} \quad , \quad R^a\gamma\psi = \begin{pmatrix} \lambda\psi^a \\ [0]_{9x1} \end{pmatrix} \tag{24}$$

$$R^{ab}\psi = \begin{pmatrix} \psi^{ab} \\ [0]_{9x1} \end{pmatrix} \quad , \quad R^{ab}\gamma\psi = \begin{pmatrix} (1 - \lambda)\psi^{ab} \\ [0]_{9x1} \end{pmatrix} \quad , \quad a = 0, 1, 2, 3.\tag{25}$$

As it will be clear below, these choices of $\lambda = 0$ correspond to the Maxwell's equations and $\lambda = 1$ to a topological field [21]. The relations among the ψ field components now are

$$\begin{aligned}\lambda\psi^a &= i\partial_b\psi^{ab}, \\ i(1-\lambda)\psi^{ab} &= \partial^a\psi^b - \partial^b\psi^a.\end{aligned}\tag{26}$$

If the matrix Φ is now written as

$$\Phi = \begin{pmatrix} [\phi^a]_{4 \times 1} \\ [\phi^{ab}]_{6 \times 1} \end{pmatrix}\tag{27}$$

the gauge transformation (20) becomes

$$\psi'^a = \psi^a + (1-\lambda)\phi^a, \quad \psi'^{ab} = \psi^{ab} + \lambda\phi^{ab},\tag{28}$$

while (21) gives

$$\lambda\partial_b\phi^{ab} = 0, \quad (1-\lambda)(\partial^a\phi^b - \partial^b\phi^a) = 0.\tag{29}$$

From (26) we see that the $\lambda = 1$ case corresponds to a topological field propagating no degree of freedom, which will not be considered here. On the other hand, the $\lambda = 0$ case corresponds to the Maxwell electromagnetic field, what can be promptly realized from (26) by setting

$$\psi^a = \frac{1}{\sqrt{2}} A^a.\tag{30}$$

The gauge transformation (28-29) now reads

$$A'^a = A^a + \phi^a, \quad \phi^a = \partial^a\Lambda(x),\tag{31}$$

where $\Lambda(x)$ is some arbitrary function of space-time coordinates. This is the usual $U(1)$ gauge invariance.

Finally, taking the explicit form of ψ into the DKP Lagrangian density (1) results in the Maxwell's one,

$$\mathcal{L}_{\mathcal{M}_{s=1}} = -\frac{1}{4} F_{ab}F^{ab},\tag{32}$$

$$F_{ab} = \partial_a A_b - \partial_b A_a.\tag{33}$$

3 Transition to Riemann-Cartan space-times

We can do the transition from the Minkowski space-time, \mathcal{M}^4 , to the Riemann-Cartan one, \mathcal{U}^4 , through the formalism of *tetrads* and applying the standard form of the minimal coupling procedure². From now on the Latin space-time indexes a, b, \dots will refer to the Minkowski space-time $\mathcal{M}^4(x)$, which now is tangent to the Riemann-Cartan space-time \mathcal{U}^4 at the point x , whose coordinates will be labeled by the Greek letters.

The minimally coupled massless DKP lagrangian becomes

$$\mathcal{L}_{\mathcal{U}} = \sqrt{-g} [i\bar{\psi}\gamma\beta^\mu\nabla_\mu\psi - i\nabla_\mu\bar{\psi}\beta^\mu\gamma\psi - \bar{\psi}\gamma\psi],\tag{34}$$

²By minimal coupling we mean the change from usual derivatives to covariant ones as stated in [25, 26, 27]. This is usually referred as the “comma to semicolon” rule.

where g is the determinant of the Riemann-Cartan metric tensor $g^{\mu\nu}$ and the matrices $\beta^\mu = \beta^\mu(x)$ are defined through contraction with the tetrad (or *vierbein*) fields $e^\mu{}_a(x)$, i.e., $\beta^\mu = e^\mu{}_a\beta^a$. These matrices satisfy the generalized DKP algebra³

$$\beta^\mu\beta^\nu\beta^\alpha + \beta^\alpha\beta^\nu\beta^\mu = \beta^\mu g^{\nu\alpha} + \beta^\alpha g^{\nu\mu}, \quad (35)$$

$$\beta^\mu\gamma + \gamma\beta^\mu = \beta^\mu \quad \text{and} \quad \gamma^2 = \gamma. \quad (36)$$

In the above lagrangian ∇ is the Einstein-Cartan covariant derivative associated with a connection $\Gamma_{\mu\nu}^\alpha$, whose antisymmetric part defines the *torsion tensor* $Q_{\mu\nu}^\alpha$, i.e.,

$$Q_{\mu\nu}^\alpha = \frac{1}{2} (\Gamma_{\mu\nu}^\alpha - \Gamma_{\nu\mu}^\alpha).$$

The *contorsion tensor* $K_{\mu\nu}^\alpha$ is defined as

$$K_{\mu\nu}^\alpha = \overset{r}{\Gamma}_{\mu\nu}^\alpha - \Gamma_{\mu\nu}^\alpha = -Q_{\mu\nu}^\alpha + Q_{\mu}^\alpha{}_\nu + Q_{\nu}^\alpha{}_\mu,$$

where $\overset{r}{\Gamma}_{\mu\nu}^\alpha$ are the Christoffel symbols, or the Riemannian part of the connection $\Gamma_{\mu\nu}^\alpha$. The covariant derivatives of DKP field are given by

$$\nabla_\mu\psi = \partial_\mu\psi + \frac{1}{2}\omega_{\mu ab}S^{ab}\psi, \quad \nabla_\mu\bar{\psi} = \partial_\mu\bar{\psi} - \frac{1}{2}\omega_{\mu ab}\bar{\psi}S^{ab},$$

where $S^{ab} = [\beta^a, \beta^b]$ and $\omega_{\mu ab}$ is the spin connection [19]. In the Einstein-Cartan theory the spin connection can be written in terms of the affine connection and the tetrad field as [25]

$$\omega_\mu{}^{ab} = \gamma_\mu{}^{ab} - K_\mu{}^{ba}, \quad (37)$$

where the term $\gamma_\mu{}^{ab}$ in (37) is referred to as the *riemannian part* of the spin connection.⁴

From the Lagrangian density (34) we obtain the generalized massless DKP equation in a Riemann-Cartan space-time

$$i\beta^\mu\nabla_\mu\psi + iK_{\mu\nu}^\mu\beta^\nu\gamma\psi - \gamma\psi = 0.$$

Similarly to what happens with the massive DKP and Dirac fields, the minimal coupling procedure on the massless lagrangian density leads to a *non-minimally* coupled equation of motion [25, 19]. This equation is invariant under the gauge transformation (5) if, and only if, Φ satisfies

$$\beta^\mu\nabla_\mu(1-\gamma)\Phi = \gamma\beta^\mu\nabla_\mu\Phi = 0, \quad (38)$$

which will be assumed from now on. This condition is nothing more than a generalization of condition (6).

³We denote by η^{ab} the (constant) metric tensor of $\mathcal{M}^4(x)$.

⁴This term is given by

$$\gamma_\mu{}^{ab} = -\gamma_\mu{}^{ba} = e_{\mu i} (C^{abi} - C^{bia} - C^{iab}),$$

where C^{abi} are the Ricci rotation coefficients

$$C_{ab}{}^i = e^\mu{}_a(x) e^\nu{}_b(x) \partial_{[\mu} e_{\nu]}{}^i.$$

The brackets in this expression denote symmetrization of the enclosed indexes.

3.1 Spin 0 sector

The Riemann-Cartan spin 0 projectors P and $P^\mu (= e^\mu{}_a P^a)$ are such that $P\psi$ and $P^\mu\psi$ transform, respectively, as a scalar and a vector in the Riemann-Cartan space-time [19]. Following the same procedure done in the Minkowski case we obtain the equation of motion for the field $P\psi$ in a Riemann-Cartan space-time

$$(\nabla_\mu + K_{\alpha\mu}{}^\alpha)(\nabla^\mu + K_\beta{}^\mu{}^\beta)\gamma(P\psi) = 0, \quad (39)$$

as well as the gauge transformation

$$P\psi' = P\psi + P(1 - \gamma)\Phi \quad , \quad P^\mu\psi' = P^\mu\psi + P^\mu(1 - \gamma)\Phi. \quad (40)$$

It is straightforward to verify that equation (39) is invariant under the above gauge transformation.

We turn to the specific representation mentioned in the previous section 2.1, and restrict our attention to the $\lambda = 0$ case only, which in the Minkowski space-time represents the usual massless Klein-Gordon field. Here the relations among the ψ components are

$$\psi^\mu = i\nabla^\mu\varphi = i\partial^\mu\varphi,$$

such that equation (39) now reads

$$(\nabla_\mu + K_{\alpha\mu}{}^\alpha)\partial^\mu\varphi = \nabla_\mu^\Gamma\partial^\mu\varphi = 0.$$

Thus, contrary to what happens in the massive case [19], we conclude that *the spin 0 sector of the massless DKP field does not interact with the space-time torsion*. With Φ given by (15) the gauge transformation (40) now reads

$$\varphi' = \varphi + \varphi_\Phi \quad , \quad \psi'^\mu = \psi^\mu,$$

while condition (38) becomes $\nabla_\mu\varphi_\Phi = \partial_\mu\varphi_\Phi = 0$, i.e., φ_Φ must be a constant.

Finally, in this representation the lagrangian density for the spin 0 sector of the massless DKP field in Riemann-Cartan space-time reduces to the usual one obtained from the Minkowski Klein-Gordon lagrangian density, i.e.,

$$\mathcal{L}_{\mathcal{RC}_{s=0}} = \sqrt{-g} \partial^\mu\varphi^* \partial_\mu\varphi. \quad (41)$$

3.2 Spin 1 sector

The Riemann-Cartan spin 1 projectors $R^\mu (= e^\mu{}_a R^a)$ and $R^{\mu\nu} (= e^\mu{}_a e^\nu{}_b R^{ab})$ are such that $R^\mu\psi$ and $R^{\mu\nu}\psi$ transform respectively as a vector and a second rank tensor in the Riemann-Cartan space-time [20]. Following the same steps of the previous sections we obtain the equation of motion for the field $R^\mu\psi$

$$(\nabla_\beta + K_{\sigma\beta}{}^\sigma)(\nabla_\alpha + K_{\sigma\alpha}{}^\sigma)\gamma T^{\alpha\beta\mu} = 0, \quad (42)$$

where $T^{\alpha\beta\mu} = g^{\alpha\beta}(R^\mu\psi) - g^{\alpha\mu}(R^\beta\psi)$, and the gauge transformation

$$R^\mu\psi = R^\mu\psi + R^\mu(1 - \gamma)\Phi \quad , \quad R^{\mu\nu}\psi' = R^{\mu\nu}\psi + R^{\mu\nu}(1 - \gamma)\Phi, \quad (43)$$

together with the conditions

$$\begin{aligned} \nabla_\nu R^{\mu\nu}(1 - \gamma)\Phi &= 0, \\ \nabla^\mu R^\nu(1 - \gamma)\Phi - \nabla^\nu R^\mu(1 - \gamma)\Phi &= 0. \end{aligned} \quad (44)$$

Again it is straightforward to show that equation (42) is invariant under the above gauge transformation.

Working with an explicit representation, as in section 2.2, and setting $\lambda = 0$, which in the Minkowski space-time results in the electromagnetic field, we get the following relations among ψ components

$$\begin{aligned} i\psi^{\mu\nu} &= \nabla^\mu\psi^\nu - \nabla^\nu\psi^\mu, \\ (\nabla_\nu + K_{\sigma\nu}^\sigma) \psi^{\mu\nu} &= 0, \end{aligned} \tag{45}$$

which leads to the equation of motion for the spin 1 sector of the massless DKP field in a Riemann-Cartan space-time

$$(\nabla_\nu + K_{\sigma\nu}^\sigma)(\nabla^\mu\psi^\nu - \nabla^\nu\psi^\mu) = 0.$$

The gauge transformation (43) now reads

$$\psi'^\mu = \psi^\mu + \Phi^\mu, \quad \psi'^{\mu\nu} = \psi^{\mu\nu}$$

and the condition (44) becomes

$$\nabla^\mu\Phi^\nu - \nabla^\nu\Phi^\mu = 0,$$

or, explicitly,

$$\partial_\mu\Phi_\nu - \partial_\nu\Phi_\mu - 2Q_{\mu\nu}^\alpha\Phi_\alpha = 0. \tag{46}$$

If $Q_{\mu\nu}^\alpha \equiv 0$ (an identically vanishing torsion) the gauge transformation above reduces to the usual $U(1)$ gauge transformation of the electromagnetic field, i.e.,

$$\Phi_\mu = \partial_\mu\Lambda,$$

where Λ is an arbitrary function of the space-time coordinates. If this is not so, *the condition (46), in general, breaks the usual $U(1)$ local gauge invariance* (by the way, the *global* one is still preserved). On the other hand, we can find classes of solutions of (46), in terms of torsion, which preserve the local gauge symmetry.

With these results, the spin 1 sector of the massless DKP lagrangian density in Riemann-Cartan space-time becomes

$$\mathcal{L}_{RC_{s=1}} = \sqrt{-g} \left(-\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + F_{\mu\nu}Q^{\mu\nu}{}_\sigma A^\sigma - Q_{\mu\nu}{}^\rho Q^{\mu\nu}{}_\sigma A_\rho A^\sigma \right),$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

with $A^\mu = \sqrt{2}\psi^\mu$ being a real vector field. Clearly, the terms breaking the $U(1)$ local gauge invariance are those which couple the torsion to the massless spin 1 field.

4 Remarks and Conclusions

In this work we considered the massless version of DKP theory obtained from an explicitly gauge invariant lagrangian density. We generalized the theory from Minkowski to Riemann-Cartan space-times through the tetrad formalism and applying the minimal coupling procedure in its standard form, i.e., using the “comma to semicolon rule”. Through the use of Umezawa’s projectors we analyzed the question of the gauge invariance, both in the spin 0 and spin 1 sectors of the theory.

We found that in the spin 0 sector the usual global gauge invariance is preserved in the transition to Riemann-Cartan space-times (which includes the Riemannian space-time of general relativity). Moreover, and differently to what was observed in the massive case [19], we found that the massless spin 0 DKP field does not couple with the space-time torsion. Summarizing, *the spin 0 sector of this massless DKP theory is completely equivalent to the Klein-Gordon one.*

On the other hand, the spin 1 sector of the theory is invariant under $U(1)$ local gauge transformations, both in the Minkowski and in the Riemannian space-times, and in these cases it is completely equivalent to the Maxwell electromagnetic theory. Nevertheless, in Riemann-Cartan space-times with a nonvanishing torsion there is, in general, a breaking of the local $U(1)$ gauge symmetry; although it can be found torsion solutions which will preserve the local gauge symmetry. These results are similar to those appearing in the context of Maxwell electromagnetic theory in the presence of torsion, to which there are several approaches in the literature. For instance, in reference [28] a modified form of the local gauge invariance is presented, in the context of the minimal coupling procedure, which allows interaction of the electromagnetic field with a dynamical torsion. In reference [29] a theory is presented in which the torsion interacts with the electromagnetic field without modifying the form of local gauge invariance, but at the cost of introducing a semi-minimal photon-torsion coupling, justified on the grounds of physical reasonableness. Alternatively, in reference [30], the authors propose a redefinition of the minimal coupling rule, such that the gauge invariance is required from the beginning; and in [31] the minimal coupling procedure is abandoned in favor of an axiomatic construction based on conservation laws.

As further developments on the massless case, it seems interesting to study the quantization of spin 1 sector of the massless DKP theory by exploring the local gauge symmetry. There exist the latent possibility of consistent quantization in Riemann-Cartan space-time. Besides that, it seems interesting to study the above subjects in the context of another (nonequivalent) theories for massless DKP fields, as well as the conformal properties of these fields on Riemann-Cartan space-times. These questions are presently under our consideration.

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